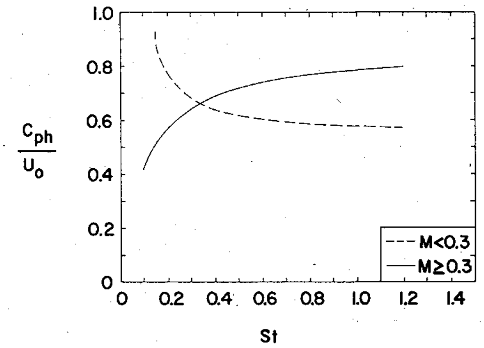


**Fig. 3 Nondimensional axial wave speed ($Cph/Uo$) variation with Strouhal number.**

wavenumber-frequency relationship is independent of all of the aforementioned quantities.

Figure 2 contains the remaining wavenumber-frequency data where the Mach number is less than 0.3. The works of Crow and Champagne and Chan compare favorably with a curve fit (for $n=0$) showing that the wavenumber-frequency relationship follows the form: $krD = -0.7435 + 11.714St$. Comparing the slope of this curve fit to the previous one shows that there was a definite difference in the two different Mach number ranges. For both of the incompressible studies, the $n=0$ mode was excited by a speaker in the plenum. For the two higher modes studied by Chan, six acoustic drivers were located around the azimuth of the jet at the exit. The offset of the $n=0$ data from the two higher modes may reflect the different excitation techniques used since, in the compressible data, many different modal compositions all maintained the same axial wavenumber-frequency relationship.

Figure 3 shows the nondimensional axial wave speed ($Cph/Uo$) for the different frequencies as calculated from the curve fits using $Cph/Uo = 2\pi St/krD$. For the compressible flow data, the wave speed increases with increasing frequency and asympotes to a maximum value of about 87% of the exit velocity of the jet ($Uo$). For the incompresssible flow, a completely different trend is observed. The speed decreases with increasing frequency and asympotes to a value of about 54% of the exit velocity. In both cases, the wave systems are dispersive, with wave speed varying with frequency.

The data presented show that when the Mach number of an axisymmetric jet is above 0.3, the wavenumber-frequency relationship is independent of Reynolds number, Mach number, initial shear layer condition, and azimuthal modal content. Also, in this Mach number range, the coherent structures move downstream with increasing speed as the frequency increases. For $M<0.3$, the axial wavenumber-frequency relationship is different than for the other jets and exhibits some dependence upon the modal content. These incompressible jets possess coherent structures whose axial wave speed decreases with increasing frequency and is dependent upon the initial shear layer condition. Further investigation is required to determine if this change in the nature of the coherent structure is due to compressibility effects or to other experimental effects such as the level of artificial excitation used.

## Acknowledgments

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# Computations of Unsteady Transonic Cascade Flows

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## Introduction

IN an oscillating cascade there is by definition a fundamental periodicity that occurs every $2N$ blades ($2N$ is the number of blades in the compressor row). The unsteady flow at each blade will have a periodic boundary condition, as in steady flow, but will lag by a phase angle of $(p/N)\pi$ in relation to the neighboring blade, where $p$ is an integer less than or equal to $N$ whose value is determined as part of the flutter solution. In a nonlinear transonic numerical scheme the choice is between computing the entire $2N$ blade sequence with the usual periodic boundary conditions at the extremities or computing a three-blade cascade problem for each specified value of $p$. These numerical calculations are computationally expensive and it is desirable to reduce the overall cost of a flutter calculation. Both of these choices involve a large amount of computer time for practical cases and, in the case of the first choice, a major development of a computer code. However, it is possible to devise a simpler approach to the problem.

The basic idea of this Note is to devise an elementary problem in which only one blade is in motion, the others being fixed; the motion may be any general time-dependent function. This removes the problem of computing the flow for each blade phase angle. This elementary problem is solved for a particular moving blade and the functional form of the velocity potential for both space and time is then known. Because of periodicity in both space and time, these

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elementary solutions can be superimposed with reparameterized space and time variables, so that the sum of the solutions satisfies both the basic differential equation and the correct boundary conditions on every blade. Although the most important application of this superposition principle is the development of the correct linearization of the transonic flow equations with discontinuities, these equations are much too complicated for an illustration of the superposition technique. Hence, in the following only a simple, subsonic problem is examined. The general theory is directly extendable to discontinuous transonic flows using the strained coordinate theory of Nixon[1] in which unsteady flow perturbations with moving shock waves can be represented by two equations linear in the strained coordinates thus allowing superposition.

## Analysis

Consider the cascade of $2N$ blades shown in Fig. 1, where blade $J+N$ and blade $J-N$ are identified. The equation for the perturbation velocity potential, $\phi(\bar{x},t)$, due to a time-dependent behavior of the blades, will be linear and can be represented as

$$L(\bar{x},t)\phi=0 \tag{1}$$

Here, $\bar{x}$ is a general vector coordinate centered on the blade of interest, $t$ the time, and $L(\bar{x},t)$ a differential operator with variable coefficients. These coefficients arise from the mean steady flow solution about which the flow is perturbed, are functions only of $\bar{x}$, and are periodic in space with period of $2N$ blades. The time $t$ occurs only through the inclusion of time derivatives. Such an operator occurs in the subsonic flow analysis of Verdon and Caspar[2] who use the equation

$$\left\{\left(\frac{\partial}{\partial t}+\nabla\Phi\cdot\nabla\right)\left(\frac{\partial}{\partial t}+\nabla\Phi\cdot\nabla\right)+(\gamma-1)\nabla\Phi\left(\frac{\partial}{\partial t}+\nabla\Phi\cdot\nabla\right)\right.$$
$$\left.+\nabla\left[\frac{\nabla\Phi\cdot\nabla\Phi}{2}\right]\cdot\nabla-a^2\nabla^2\right\}\phi=0$$

where $\Phi$ is the steady-state potential. For a staggered cascade the upstream and downstream boundary conditions will depend on the particular region of the flow that is being considered, for example, the far upstream boundary $D_{uj}$ and the far downstream boundary $D_{Dj}$ shown in Fig. 1. The boundary conditions will depend on the steady far-field velocity, which has spatial periodicity, and on the phase of the waves produced by the oscillating cascade. Since the individual blade flows are identical except for a phase lag of $jt_0$, it follows that the far-field boundary conditions are

$$\phi_u(\bar{x}_u,t)=g_u(\bar{x}_u,t_j)\ \text{ on } D_{uj},\quad j=-N,N$$
$$\phi_D(\bar{x}_D,t)=g_D(\bar{x}_C,t_j)\ \text{ on } D_{Dj},\quad j=-N,N \tag{2}$$

where $D_{uj}$ and $D_{Dj}$ denote the upstream and downstream boundaries, respectively, and $g_u$ and $g_D$ are the corresponding spatially asymptotic values of $\phi(\bar{x},t)$. The time $t_j$ is given by $t_j=t-jt_0$ where $t_0$ is the interblade phase lag. The tangential boundary conditions are

$$\left.\begin{array}{l} v_j^+(\bar{x},t)=f_j^+(\bar{x},t_j) \\ v_j^-(\bar{x},t)=f_j^-(\bar{x},t_j)\end{array}\right\}\ \text{on } S_j;\ j=-N,N \tag{3}$$

where $v_j$ is the normal velocity component, $f_j$ is a function of the specified perturbation of blade geometry, and $S_j$ denotes the location of the $j$th blade surface. The $\pm$ signs denote conditions on the upper and lower surface of the blades, respectively. The effect of the interblade phase lag is seen in Eqs. (2) and (3) where the potentials $\phi_u$, $\phi_D$ or the velocity of the $j$th blade at time $t$ are given in terms of a function at a time $t_j$. For blade 0, periodicity can be applied on $j=\pm N$. In

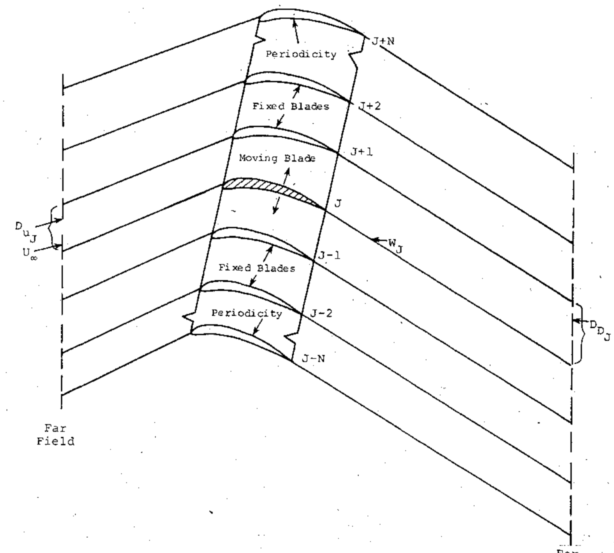


**Fig. 1 Cascade problem.**

addition to the above boundary conditions the wake condition is

$$\Delta C_p(\bar{x},t)=0\ \text{ on } w_j,\ j=-N,N \tag{4}$$

where $\Delta C_p$ is the pressure jump across the wake of the $j$th blade $w_j$ and is a function of $\phi(\bar{x},t)$. Now if the principle of superposition holds, then a variable $\phi_j(\bar{x}_j,t)$ can be introduced such that

$$\phi(\bar{x},t)=\sum_{j=-N}^{N}\phi_j(\bar{x}_j,t) \tag{5}$$

where

$$\bar{x}_j=\bar{x}-\tilde{x}_j$$

and $\tilde{x}_j$ is the location of the general coordinate system centered on the $j$th blade and $x_0\equiv\bar{x}$. Assume that $\phi_j$ is such that

$$L(\bar{x},t)\phi_j=0 \tag{6}$$

It is obvious that if Eq. (6) is used with Eq. (5), then Eq. (1) is recovered.

The operator $L(\bar{x},t)$ is a differential operator with coefficients that are functions of the steady flow over a cascade and hence $L(\bar{x},t)$ is periodic in the spatial dimensions. Thus $L(\bar{x},t)$ is the same no matter on which blade the coordinate system is centered. Hence

$$L(\bar{x},t)=L(\bar{x}_j,t) \tag{7}$$

Furthermore, since $t$ appears only in the operator through a derivative, the variable $t$ can be replaced by $t_j$. Thus

$$L(\bar{x},t)=L(\bar{x}_j,t_j) \tag{8}$$

Using Eqs. (6) and (8) then gives

$$L(\bar{x}_j,t_j)\phi_j=0 \tag{9}$$

The next task is to determine suitable boundary conditions. Let

$$\begin{array}{lll}\phi_j(\bar{x}_u,t_j)=g_u(\bar{x}_u,t_j) & \text{on } D_{uj} & \\ \phi_j(\bar{x}_D,t_j)=g_D(\bar{x}_D,t_j) & \text{on } D_{Dj} & \\ \phi_j(\bar{x}_u,t_j)=0 & \text{on } D_{uk}, & k\neq j \\ \phi_j(\bar{x}_D,t_j)=0 & \text{on } D_{Dk}, & k\neq j\end{array} \tag{10}$$

$$\left.\begin{aligned} v_j^+(\bar{x}_j,t_j) &= y'_{s+}(\bar{x}_j,t_j) \\ v_j^-(\bar{x}_j,t_j) &= y'_{s-}(\bar{x}_j,t_j) \end{aligned}\right\} \text{ on } S_j$$

$$v_j^+[(\bar{x}_j-\bar{x}_k),t_j] = v_j^-[(\bar{x}_j-\bar{x}_k),t_j] = 0 \quad \text{on } S_{j+k} \qquad k=1,N-1$$

$$v_j^+[(\bar{x}_j-\bar{x}_k),t_j] = v_j^-[(\bar{x}_j+\bar{x}_\ell),t_j] = 0 \quad \text{on } S_{j-\ell} \qquad \ell=1,N-1$$

$$\Delta C_{pj}(\bar{x}_j,t_j) = 0 \text{ on } w_j \tag{11}$$

where

$$C_p(\bar{x},t) = \sum_{-N}^{N} C_{pj}(\bar{x}_j,t) \tag{12}$$

and $y=y'_{s+}(\bar{x}_j,t_j)$ and $y=y'_{s-}(\bar{x}_j,t_j)$ denote the slopes on the upper and lower surfaces of the $j$th blade, respectively. It should be noted that in the $(\bar{x}_j,t_j)$ coordinate system each of elementary problems for $\phi_j$ is identical.

Periodicity is applied on the $j=\pm N$ blades and their wakes. The wake boundary condition is given by

$$\Delta C_{p_j}[(\bar{x}_j-\bar{x}_k),t_j] = 0 \text{ on } w_{j+k} \quad k=1,N-1$$

$$\Delta C_{p_j}[(\bar{x}_j+x_\ell),t_j] = 0 \text{ on } w_{j-\ell} \quad \ell=1,N-1 \tag{13}$$

This is equivalent to keeping all blades stationary except the $j$th blade and allows the relevant wave transmission through each blade wake. Note that the time is always $t_j$, the time associated with the $j$th blade. No interblade phase lag is required at this stage.

When $\phi_j$ are summed, together with the boundary conditions, the problem defined by Eqs. (5) and (10-13) is identical to the problem defined by Eqs. (1-4). Hence the problem for any time lag $t_0$ between blades can be constructed from the superposition of the elementary problem defined by Eqs. (9) and (11-13). The superposition mechanism is as follows.

Let the solution to the elementary problem for $j=0$ be given by $\phi_0(\bar{x},t)$. The solution for $\phi_j(\bar{x},t_j)$ is then given by

$$\phi_j(\bar{x}_j,t_j) = \phi_0(\bar{x},t) \tag{14}$$

Since the functional form of $\phi_0$ with both $\bar{x}$ and $t$ is known from the elementary solution for $j=0$, this reparameterization is trivial. The final solution for the zeroth blade is then given by Eqs. (5) and (14); thus

$$\phi(\bar{x},t) = \sum_{-N}^{N} \phi_0(\bar{x}-\bar{x}_j,\ t-jt_0) \tag{15}$$

Thus the complete time-dependent cascade flow for any interblade phase angle can be constructed by superposition of the elementary problem defined by Eqs. (5) and (10-13).

If $\phi(\bar{x},t)$ and its derivatives are continuous, a similar relation to Eq. (15) can be constructed for the pressure coefficient $C_p(\bar{x},t)$. In addition, similar formulas can be derived for the lift and moment coefficients.

The idea discussed above can be extended to discontinuous transonic flows using the method of strained coordinates.[1] Results are given by Kerlick and Nixon.[3]

## Conclusion

The main contribution to the computation time for an unsteady calculation of cascade flutter in a transonic flow is the need to repeat the calculation for a range of interblade phase angles. The present analysis shows how this problem can be eliminated by a judicious choice of elementary solutions.

## Acknowledgment

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# Finite Element Modeling Techniques for Constrained Layer Damping

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## Introduction

CONSTRAINED layer damping treatments are used to suppress noise and vibration in complex structural systems. These treatments provide effective suppression by dissipating energy in a soft, heavily damped, viscoelastic core sandwiched between the two face sheets of a composite panel in flexure.

A number of authors have considered analytical techniques to predict the performance of constrained layer damped beams and plates. Ross et al.[1] developed a fourth-order theory for simply supported composite beams which have a complex modulus core. Rao[2] formulated a sixth-order theory using an energy approach, and obtained exact solutions for these composite beams which have various boundary conditions.

Johnson et al.[3] developed a three-dimensional finite element plate model using the MSC/NASTRAN computer program. The base beam and the constraining layer were modeled by plate elements. The viscoelastic core was represented by solid elements. The material properties of the viscoelastic core were represented by a complex, frequency dependent, shear modulus $G(\omega)[1+i\eta(\omega)]$. They showed that the modal strain energy method could be used to accurately and efficiently solve this problem.

The modal strain energy method uses the undamped modal characteristics. Modal loss factors are computed using the relation

$$\eta_r = \Big(\sum_e \eta_e \phi_{re}^T k_e \phi_{re}\Big) \Big/ \Big(\sum_e \phi_{re}^T k_e \phi_{re}\Big) \tag{1}$$

where $\eta_r$ is the modal loss factor for the $r$th mode, $\phi_{re}$ the undamped modal vector for the $e$th element in the $r$th mode, $\eta_e$ the loss factor for the $e$th element, and $k_e$ the stiffness matrix for the $e$th element. This method offers the following advantages for the analysis of constrained layer damped structures.

1) The method's computational costs are reasonable because undamped modal characteristics are used.

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